

**MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 11**

SPRING 2019
DR. ABDUL-RAHMAN

- (1) Let X be any continuous random variable. For any $t > 0$, prove that

$$\mathbb{P}(X > y) \leq e^{-ty} \mathbb{E}(e^{tX}).$$

- (2) A basketball player makes 80% of his free throws on the average. The player attempts free throws repeatedly until he makes 25. What is the probability that at least 29 throws will be necessary?
- (3) Suppose 1% of all screws made by a machine are defective. We are interested in the probability that a batch of 225 screws has at most one defective screw. Compute this probability by
- (a) Poisson approximation.
 - (b) normal approximation.

Note that the exact value of this probability is 0.34106. Which approximation works better when p is small and n is large?

- (4) Let X_1, X_2, \dots, X_n be a sequence of independent standard normal random variables. Let $S_n = X_1^2 + \dots + X_n^2$. Find the mean value and variance of S_n . Describe the normal approximation to S_n . Use it to estimate

$$\mathbb{P}(S_n \leq n + \sqrt{2n}).$$

Note: S_n is said to have χ^2 -distribution with n degrees of freedom.